Performance Analysis of ARQ Schemes for Wireless Networks

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Abstract— We present a unified method to compute the exact performance of ARQ schemes for wireless networks. A wireless channel is modeled here as a stochastic sequential machine. Accordingly, we propose a channel state ordered representation of the performance provided by an underlying block code. Based on proper events introduced to assess reliability, throughput, and delay performance of an ARQ scheme, we employ a discrete system model to obtain a matrix description for ARQ schemes used on wireless channels. Then, closed formulae are developed for all examined performance characteristics. Numerical results are given for a wireless ATM system.

I. Introduction

Wireless networks can be characterized as either wireless local area networks mainly designed for operating in an indoor environment [1][2] or wide area networks [3]. The asynchronous transfer mode (ATM) is expected to play an important role in future generation wireless networks as well [4]. Wireless networks are supposed to support high quality voice, video, and high speed data services. Therefore, one of the challenges when developing a wireless network for highly demanding applications is to combat the complex error mechanisms of the radio path to guarantee high quality services. To support the services provided by a wireless network, sophisticated error control techniques have to be used.

In order to maintain adequate system performance, block codes are used in combination with a retransmission strategy establishing an automatic repeat request (ARQ) scheme. The fundamental ARQ schemes are classified into stop-andwait (SW), go-back-N (GBN), and selective repeat (SR) ARQ. Clearly, a performance analysis of an ARQ scheme has to include a suitable channel model.

Fading channels such as in wireless networks cause nonindependent channel errors, so that a memoryless channel is not a realistic model. Because of statistical dependencies between errors, a performance analysis of an ARQ scheme in a wireless channel requires more computational effort as in the case of a simple memoryless channel. Contributions dealing with ARQ schemes in nonindependent errors are very limited and focus on specific types of channel errors [5]. Assumptions taken to simplify the mathematical tractability often result in a model close to a memoryless setting, e.g. symmetrically dependent errors [6].

This paper presents an analytical method for evaluating performance of ARQ schemes in nonindependent errors, i.e. channels with memory. An algebraically decoded linear block code is utilized for error detection. Correspond-

ing to the used stochastic finite-state channel model, performance of the employed block code is structured into a channel state ordered representation as well. By defining suitable events to be assessed, using automata theory for discrete system modeling and the concept of matrix functions, the proposed matrix approach leads to closed formulae for reliability, throughput, and delay. The unified method is applicable for any generative discrete channel model and gives exact results.

The paper is organized as follows. In Section II, we present a discrete system model for ARQ schemes in wireless channels. On this basis, closed formulae for reliability, throughput, and delay performance are developed in Section III, IV, and V, respectively. Numerical results are given in Section VI and Section VII concludes the paper.

II. MODELING OF ARQ SCHEMES IN WIRELESS CHANNELS

In this paper, a wireless channel is modeled as a stochastic sequential machine (SSM). We make use of the stationary probability distribution

$$\sigma_0 = [\sigma_{s_0}^{(0)}, \sigma_{s_1}^{(0)}, \cdots, \sigma_{s_{S-1}}^{(0)}] \tag{1}$$

on the S channel states s and the state transition matrix

$$\mathbf{D} = [p(s'|s)]_{S \times S}, \qquad (2)$$

where p(s'|s) denotes the probability when the channel is in state s that the subsequent state will be s'.

A block code is utilized for error detection and is algebraically decoded. Usually, performance of a block coded system without any retransmission strategy is characterized in terms of

 P_c : Probability of correct decoding,

 P_e : Probability of erroneous decoding,

 P_d : Probability of error detection,

with

$$P_c + P_e + P_d = 1. ag{3}$$

Performance of a block code can be further classified by the probability P_D of decoding and the probability P_f of an error. We have

$$P_D = P_c + P_e \quad \text{and} \quad P_f = P_e + P_d. \tag{4}$$

To assess the performance of an ARQ scheme in a wireless channel, we assume the performance of a block code is given in a channel state ordered representation, e.g. $p_c(s^{(n)}|s^{(0)})$, where $s^{(0)}$ denotes an initial channel state and $s^{(n)}$ is the channel state reached after transmitting a packet of length n. This requirement can be satisfied by using the method presented in [7]. The various conditional probabilities are taken as elements of matrix probabilities

$$P_{c} = [p_{c}(s^{(n)}|s^{(0)})]_{S\times S},$$

$$P_{e} = [p_{e}(s^{(n)}|s^{(0)})]_{S\times S},$$

$$P_{d} = [p_{d}(s^{(n)}|s^{(0)})]_{S\times S},$$
(5)

which are interconnected via [8]

$$\mathbf{P}_c + \mathbf{P}_e + \mathbf{P}_d = \mathbf{D}^n. \tag{6}$$

A matrix equivalent to (4) can be formally written as

$$\mathbf{P}_D = \mathbf{P}_c + \mathbf{P}_e = [p_D(s^{(n)}|s^{(0)})]_{S \times S}, \tag{7}$$

$$\mathbf{P}_f = \mathbf{P}_e + \mathbf{P}_d = [p_f(s^{(n)}|s^{(0)})]_{S \times S}.$$
 (8)

With this approach, it is possible to include the channel state transitions during the round-trip delay by means of the state transition matrix. In doing so, the elementary events to be considered are the decoding outcomes after the ν^{th} transmission of a packet and the number of state transitions occurring between consecutive retransmissions of that packet. We use the notation:

 c_{ν} : correct decoding after the ν^{th} transmission,

 e_{ν} : erroneous decoding after the ν^{th} transmission,

$$d_{\nu}$$
: error detection after the ν^{th} transmission, (9)

 $s_{\nu}^{m}:m$ channel state transitions between the ν^{th} and $(\nu+1)^{th}$ transmission,

where m represents round-trip delay in bits. Let δ be the data rate, τ the round-trip delay time, n the code or packet length, and let the operator $\lceil x \rceil$ denote the smallest integer equal or greater than x. Then, the parameter m is given by

$$m = \begin{cases} \lceil \delta \cdot \tau \rceil & \text{for SW,} \\ \lceil \frac{\delta \cdot \tau}{n} \rceil \cdot n & \text{for SR and GBN.} \end{cases}$$
 (10)

Hereinafter, we assume a symmetric forward channel, an error-free feedback channel, and perfect synchronization. As a consequence, an analysis of the impact of the channel error process on the all-zero codeword is sufficient and neither acknowledgment nor frame losses occur.

III. RELIABILITY ANALYSIS

Let \mathcal{E} be the event that a decoder releases an erroneous packet, either after the first or after some retransmissions of that packet. We introduce a subevent

 \mathcal{E}_{ν} : Decoder releases an erroneous packet after it has been transmitted ν times.

Each subevent \mathcal{E}_{ν} is an intersection $\{a\} \cap \{b\} = \{a,b\}$ of elementary events (9) and given by

$$\mathcal{E}_{\nu} = \left\{ d_1, \ s_1^m, \ d_2, \ s_2^m, \ \cdots, \ d_{\nu-1}, \ s_{\nu-1}^m, \ e_{\nu} \right\}. \tag{11}$$

Thus, \mathcal{E} may be rewritten as union of subevents \mathcal{E}_{ν} as

$$\mathcal{E} = \bigcup_{\nu=1}^{\infty} \mathcal{E}_{\nu}. \tag{12}$$

Therefore, reliability of an ARQ scheme can be classified indirectly by way of the probability $P(\mathcal{E})$ of an undetected error, i.e. the probability of occurrence of the event \mathcal{E} . The higher the probability $P(\mathcal{E})$ is, the lower is the reliability and vice versa.

Using the concept of matrix probabilities, the stochastic nature of a subevent \mathcal{E}_{ν} can be specified analytically by

$$\mathbf{P}(\mathcal{E}_{\nu}) = (\mathbf{P}_{d}\mathbf{D}^{m})^{\nu-1} \cdot \mathbf{P}_{e}. \tag{13}$$

Because the subevents in (12) are pairwise disjoint, the matrix probability of an undetected error is simply given by the sum

$$\mathbf{P}(\mathcal{E}) = \sum_{\nu=1}^{\infty} \mathbf{P}(\mathcal{E}_{\nu}) = \sum_{\nu=1}^{\infty} (\mathbf{P}_{d} \mathbf{D}^{m})^{\nu-1} \cdot \mathbf{P}_{e}. \tag{14}$$

Since the matrix probability \mathbf{P}_D of decoding and the matrix probability \mathbf{P}_c of correct decoding can be easily computed by using the method proposed in [7], it is convenient to substitute \mathbf{P}_d and \mathbf{P}_e in (14) by

$$\mathbf{P}_d = \mathbf{D}^n - \mathbf{P}_D$$
 and $\mathbf{P}_e = \mathbf{P}_D - \mathbf{P}_c$. (15)

Performing some operations on (14), the matrix probability of undetected error of an ARQ scheme in a channel with memory can be expressed in closed form as

$$P(\mathcal{E}) = (I - D^{n+m} + P_D D^m)^{-1} (P_D - P_c).$$
 (16)

Finally, we sum over all channel states and as a result obtain the desired scalar probability of undetected error

$$P(\mathcal{E}) = \sigma_0 \cdot \mathbf{P}(\mathcal{E}) \cdot \mathbf{e}, \qquad (17)$$

where e denotes an all-one column vector.

Note that for an infinite round-trip delay and under the assumption of a regular Markov chain of channel states, each row of the state transition matrix is given by the stationary probability distribution [8]:

$$|\mathbf{D}^m|_{m\to\infty} = \mathbf{e} \cdot \boldsymbol{\sigma}_0. \tag{18}$$

Accordingly, for an infinite round-trip delay equation (17) can be rewritten as

$$P(\mathcal{E})|_{m\to\infty} = \frac{P_{e,SSM}}{P_{e,SSM} + P_{e,SSM}},$$
 (19)

where $P_{c,SSM}$ and $P_{e,SSM}$ denote the probability of correct and erroneous decoding of a block code used on a SSM,

respectively. In other words, statistical dependencies between errors within a packet are still included in the analysis whereas correlation between subsequent transmissions vanishes.

Similarly, for the special case of a binary symmetric channel (BSC) equation (17) approaches the known result [9]:

$$P(\mathcal{E})|_{S=1} = \frac{P_{e,BSC}}{P_{c,BSC} + P_{e,BSC}}, \qquad (20)$$

where $P_{c,BSC}$ and $P_{c,BSC}$ denote the probability of correct and erroneous decoding of a code on a BSC, respectively.

IV. THROUGHPUT ANALYSIS

Throughput of any ARQ scheme is dependent on the number of packets which could be transmitted during the average time spent to decode a particular packet for the first time. Hence, we focus on the random variable

T: Number of retransmissions, including the first transmission, until a packet is decoded either correctly or incorrectly for the first time.

A particular outcome ν of \mathcal{T} will be denoted as

$$\mathcal{T}_{\nu} = \{\mathcal{T} = \nu\} \tag{21}$$

and can be written as an intersection of elementary events, that is

$$\mathcal{T}_{\nu} = \{d_1, s_1^m, d_2, s_2^m, \cdots, d_{\nu-1}, s_{\nu-1}^m, D_{\nu} = c_{\nu} \cup e_{\nu}\}.$$
 (22)

In order to compute the average number of packets which could be transmitted until a packet is decoded for the first time, we introduce an expected matrix

$$\mathbf{m}_{\mathcal{T}} = \mathbf{E}\{\mathcal{T}\} = \left[E\{\mathcal{T}\}(s^{(n)}|s^{(0)})\right]_{S \times S},$$
 (23)

where the traditional concept of an expected value is generalized and structured into channel states. On that basis, we first examine throughput in channel state ordered representation and then average the result over all states:

$$m_{\mathcal{T}} = \sigma_0 \cdot \mathbf{m}_{\mathcal{T}} \cdot \mathbf{e} \quad \rightarrow \quad \eta \sim m_{\mathcal{T}}^{-1}.$$
 (24)

For that purpose, we need the matrix probabilities related to the outcomes \mathcal{T}_{ν} , $\nu = 1, \dots, \infty$. These are given by

$$\mathbf{P}(\mathcal{T}_{\nu}) = (\mathbf{P}_{d}\mathbf{D}^{m})^{\nu-1} \cdot \mathbf{P}_{D}. \tag{25}$$

Because for SW ARQ and SR ARQ only packets detected in error have to be retransmitted but for GBN ARQ also a finite number of subsequent packets must be retransmitted, we have to distinguish between

$$\mathcal{T}_{ARQ}(\mathcal{T}_{\nu}) = \begin{cases} \mathcal{T}_{\nu} & \text{for SW and SR,} \\ (\mathcal{T}_{\nu} - 1)N + 1 & \text{for GBN,} \end{cases}$$
 (26)

with subscript $ARQ \in \{SW, GBN, SR\}$, and where the parameter $N = \lceil (\delta \cdot \tau)/n \rceil - 1$ represents round-trip delay in packets. Then, we have

$$\mathbf{m}_{\mathcal{T}_{ARQ}} = \sum_{\nu=1}^{\infty} \mathcal{T}_{ARQ}(\mathcal{T}_{\nu}) \cdot (\mathbf{P}_{d}\mathbf{D}^{m})^{\nu-1} \mathbf{P}_{D}. \tag{27}$$

By manipulating (27), the expected values for the number of transmissions until a packet is decoded for the first time can be obtained in closed form as

$$m_{\mathcal{T}_{SW}} = \sigma_0 \cdot \mathbf{F}_D^{-2} \mathbf{P}_D \cdot \mathbf{e}, \tag{28}$$

$$m_{\mathcal{T}_{GBN}} = \sigma_0 \cdot \left[\mathbf{F}_D^{-1} + N \mathbf{F}_D^{-2} (\mathbf{D}^n - \mathbf{P}_D) \mathbf{D}^m \right] \mathbf{P}_D \cdot \mathbf{e},$$
 (29)

$$m_{\mathcal{T}_{SR}} = \sigma_0 \cdot \mathbf{F}_D^{-2} \mathbf{P}_D \cdot \mathbf{e}, \tag{30}$$

where the following matrix has been introduced to simplify the notation:

$$\mathbf{F}_D = \mathbf{I} - \mathbf{D}^{n+m} + \mathbf{P}_D \mathbf{D}^m. \tag{31}$$

Finally, the desired throughput of an SW ARQ, GBN ARQ, and SR ARQ scheme used on a channel with memory are given in closed form by

$$\eta_{SW} = \frac{k}{(n+\delta\cdot\tau)\cdot m_{\mathcal{T}_{SW}}}, \qquad (32)$$

$$\eta_{GBN} = \frac{k}{n \cdot m_{T_{GBN}}}, \tag{33}$$

$$\eta_{SR} = \frac{k}{n \cdot m_{T_{SR}}}.$$
 (34)

V. DELAY ANALYSIS

The delay associated with an ARQ scheme can be decomposed into queueing delay, transmission delay, and resequencing delay. In this paper, we focus on the transmission delay as result of the impact of a stochastic error process on data transmission. First, we characterize delay by the expected value of the number of transmissions until a packet is decoded correctly for the first time and the associated standard deviation. Then, we can simply include round-trip delay time and the processing time spent at the receiving end for decoding.

For that purpose, we introduce a random variable

D: Number of retransmissions, including the first transmission, until a packet is decoded correctly for the first time.

A particular outcome ν of \mathcal{D} will be denoted by

$$\mathcal{D}_{\nu} = \{\mathcal{D} = \nu\}. \tag{35}$$

We examine delay and delay variation by counting only correctly decoded packets. We assume that a genie assists the decoding process to request a retransmission for a decoded packet in the case it is released in error by the decoder.

In this context, a particular outcome of \mathcal{D} can be written as intersection of elementary events (9) and is given by

$$\mathcal{D}_{\nu} = \{ f_1, s_1^m, f_2, s_2^m, \cdots, f_{\nu-1}, s_{\nu-1}^m, c_{\nu} \},$$
 (36)

where $\{f_{\nu}\} = \{e_{\nu}\} \cup \{d_{\nu}\}$. Thus, the matrix probability of a particular outcome becomes

$$\mathbf{P}(\mathcal{D}_{\nu}) = (\mathbf{P}_f \mathbf{D}^m)^{\nu - 1} \cdot \mathbf{P}_c. \tag{37}$$

Hence, the average number of retransmissions until a packet is decoded correctly for the first time is characterized by the expected matrix

$$\mathbf{m}_{\mathcal{D}} = \mathbf{E}\{\mathcal{D}\} = \sum_{\nu=1}^{\infty} \nu \cdot (\mathbf{P}_f \mathbf{D}^m)^{\nu-1} \cdot \mathbf{P}_c.$$
 (38)

For the sake of a set of homogeneous performance formulae, we substitute

$$\mathbf{P}_f = \mathbf{D}^n - \mathbf{P}_c \tag{39}$$

in (38) and by further examining this sum, one may straightforwardly show

$$m_{\mathcal{D}} = \sigma_0 \cdot (\mathbf{I} - \mathbf{D}^{n+m} + \mathbf{P}_c \mathbf{D}^m)^{-2} \mathbf{P}_c \cdot \mathbf{e}. \tag{40}$$

Similarly, we are going to represent delay variation by the standard deviation

$$\sigma_{\mathcal{D}} = \sqrt{E\{\mathcal{D}^2\} - m_{\mathcal{D}}^2}. \tag{41}$$

Because the average number of retransmissions can be evaluated by (40), we only need to compute the moment matrix

$$\mathbf{E}(\mathcal{D}^2) = \sum_{\nu=1}^{\infty} \nu^2 \cdot (\mathbf{P}_f \mathbf{D}^m)^{\nu-1} \cdot \mathbf{P}_c. \tag{42}$$

As a result, we finally obtain the required moment

$$E(\mathcal{D}^2) = \sigma_0 \cdot (2 \mathbf{F}_c^{-3} - \mathbf{F}_c^{-2}) \mathbf{P}_c \cdot \mathbf{e}, \tag{43}$$

where

$$\mathbf{F}_c = \mathbf{I} - \mathbf{D}^{n+m} + \mathbf{P}_c \mathbf{D}^m. \tag{44}$$

Having developed closed formulae for the average number of retransmissions $m_{\mathcal{D}}$ and the standard deviation $\sigma_{\mathcal{D}}$, we can now include the physical dimension time into the analysis. We are going to classify delay time τ into three fractions:

$$\tau = \tau_{block} + \tau_{rtd} + \tau_{dec}, \tag{45}$$

where

 $\tau_{block} = n/\delta$: Delay time caused by organizing sequences of bits into packets,

 $\tau_{rtd} = m/\delta$: Round-trip delay time,

 τ_{dec} : Processing time spent for decoding.

On that basis, we simply have to multiply the average number of retransmissions and the associated standard deviation by the delay time and obtain the average delay time and delay variation:

$$m_{\tau} = \tau \cdot m_{\mathcal{D}}$$
 and $\sigma_{\tau} = \tau \cdot \sigma_{\mathcal{D}}$. (46)

VI. Numerical Results

We examine a GBN ARQ scheme used on a two-state Gilbert-Elliott channel (GEC). The GEC is defined by the transition probability P = Prob(B|G) and Q = Prob(G|B) between a 'good' state G and a 'bad' state B, respectively, as well as the bit error rates p_G and p_B in the respective state. An effective BSC is taken for comparison having the same average bit error rate \overline{p}_b as the GEC. In the following, the concept of a slow fading channel will be used to emphasize when transitions between states are rather unlikely. The corresponding GEC parameters are chosen as

$$P = 10^{-6}, \ Q = 3 \cdot 10^{-6}, \ p_G = 10^{-6}, \ \overline{p}_b = f(p_B).$$

Suppose the number of information bits taken is k = 424, which may reflect the number of bits to be protected in a wireless ATM system. A shortened cyclic code is employed for error detection and defined by the generator polynomial

$$q(x) = x^{16} + x^{15} + x^{13} + x^{9} + x^{7} + x^{6} + x^{5} + x^{3} + x + 1$$

Note that the round-trip delay m is used as parameter in all presented plots. A round-trip delay in a range up to some 100 bits may apply for an indoor wireless network whereas higher round-trip delay corresponds to mobile radio systems.

Figure 1 shows the probability $P(\mathcal{E})$ of undetected error of the (440,424) block coded GBN ARQ scheme versus average bit error rate. The scheme is used on a slow fading channel. Obviously, $P(\mathcal{E})$ approaches to the probability P_e of erroneous decoding of the underlying block code (dashed line) when we increase the round-trip delay. In other words, neglect of statistical dependencies between channel errors in adjacent packets would result in an overestimated reliability.

Figure 2 compares the throughput of the examined GBN ARQ on a slow fading channel and a BSC. When transitions between channel states are very unlikely, the decrease in throughput in high average bit error rates turns out to be more severe on the GEC compared to the performance on the effective BSC. As far as throughput is concerned, the superior reliability in high average bit error rates on the GEC leads to no improvement. This is because in the interval of high average bit error rates throughput has already dropped to almost zero.

We eventually consider average delay time versus average bit error rate. We assume a data rate of $\delta=2Mbps$, which gives $\tau_{block}=220\mu s$ and $\tau_{rtd}=0$ to 0.5s for m=0 bit to 10^6 bit. The delay fraction τ_{dec} has been neglected. Figure 3 shows the results for the average delay time. As it can be seen from the plot, for average bit error rates $\bar{p}_b \leq 10^{-3}$, the GBN ARQ provides the same constant delay on both channels. On the contrary, at high bit error rates the delay increases exponentially on the effective BSC but changes only to a higher niveau of constant average delay time for the GEC. The reason is that the probability of correct decoding on an effective BSC approaches zero for high average bit error rates. On the GEC the probability of correct decoding can be maintained non-zero because of the contribution in the good state G. Especially for the slow fading channel,

the stationary state probabilities for being in the states G and B are 75% and 25%, respectively. In addition, the average periods of time spent in G and B are 10^6 and $1/3 \cdot 10^6$ discrete time cycles, respectively.

VII. CONCLUSION

We have presented an analytical method to evaluate the exact performance of ARQ schemes in wireless channels. Corresponding to the employed stochastic finite-state channel model, performance of a block code is structured into a channel state ordered representation as well. Events associated with reliability, throughput, and delay of an ARQ scheme in a wireless channel have been introduced and are taken as a foundation for an exact performance evaluation. By using a discrete system model in combination with matrix functions, closed formulae are obtained for all examined performance characteristics. Because of the employed matrix approach, an implementation of the proposed method on a digital computer is straightforward.

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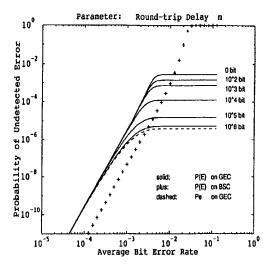


Figure 1. $P(\mathcal{E})$ of a (440, 424) GBN ARQ scheme

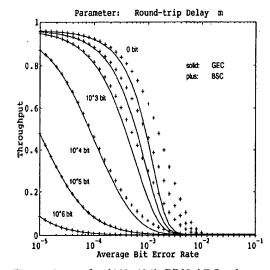


Figure 2. η of a (440, 424) GBN ARQ scheme

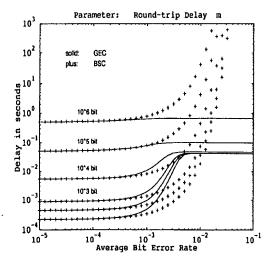


Figure 3. m_{τ} of a (440, 424) GBN ARQ scheme